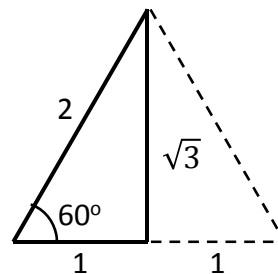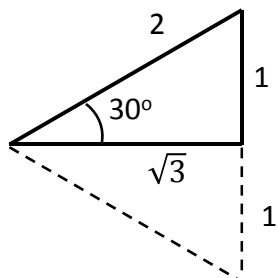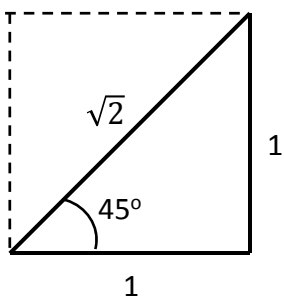**Recreate the Unit Circle:**

Question: Where do the values of the trigonometric functions for the angles on the unit circle come from?

Answer: They come from two "special" right triangles: the isosceles right triangle and the 30-60-90 right triangle.

The isosceles right triangle gives the values for 45 degrees.

Take a square of side 1 and draw a diagonal.

The hypotenuse has length $\sqrt{2}$. So

$$\sin 45° = \frac{\sqrt{2}}{2} \quad \cos 45° = \frac{\sqrt{2}}{2}$$

$$\tan 45° = 1$$

The 30-60-90 right triangle gives the values for 30 degrees and for 60 degrees.
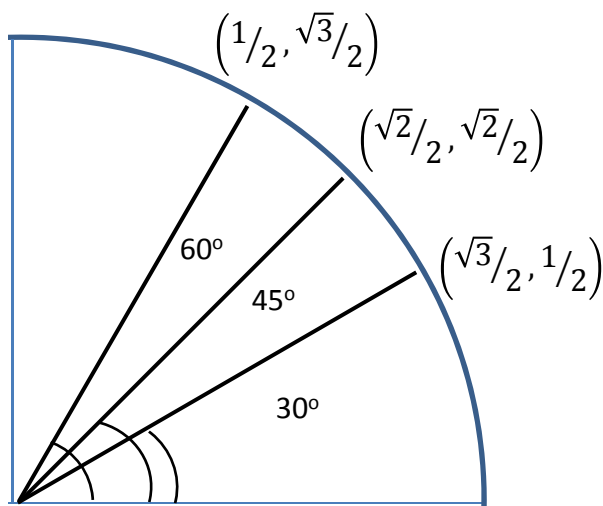
Take an equilateral triangle of side 2 and draw a perpendicular bisector.

The bisector has length $\sqrt{3}$. So

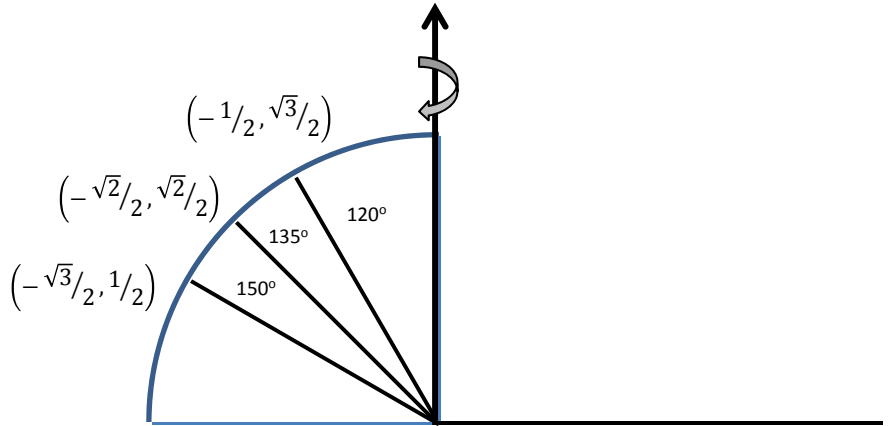$$\sin 30° = \frac{1}{2} \quad \cos 30° = \frac{\sqrt{3}}{2} \quad \tan 30° = \frac{\sqrt{3}}{3}$$

and

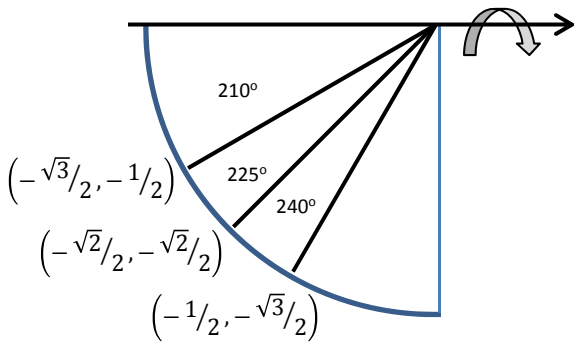$$\sin 60° = \frac{\sqrt{3}}{2} \quad \cos 60° = \frac{1}{2} \quad \tan 60° = \sqrt{3}$$

Now we can label the points $(\cos\theta, \sin\theta)$ for the first quadrant

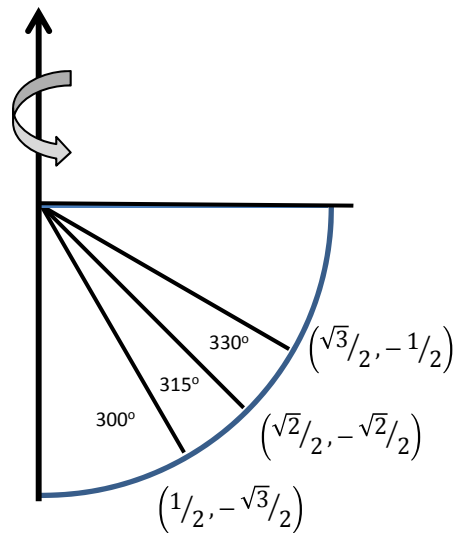To get the values for the 2nd quadrant, imagine flipping the triangles about the Y-axis

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

120°

135°

150°

We should get almost the same values as before, but the x values should be negative

210°

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ 225°

240°

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

For the third quadrant, flip about the Y-axis

Now both x and y values will be negative

330° $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

315°

300° $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

Finally, for the fourth quadrant, flip again about the Y-axis

Now only the y values will be negative

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

135° 120°   60°

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ 150°   45° $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

30°
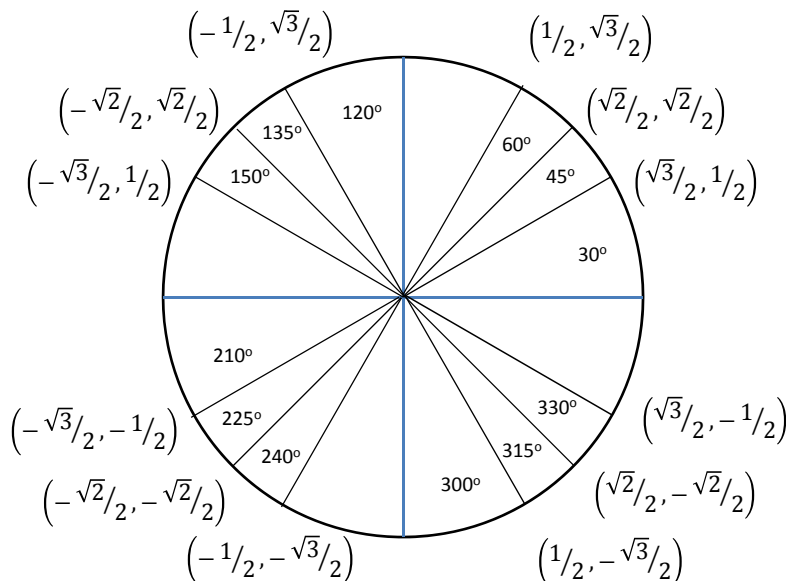
Putting it all together gives us

210°

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ 225°   330° $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

240°   315°

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ 300° $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

The Unit Circle